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# Superfluidity in $\beta$ -stable neutron star matter

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In this work we present results for pairing gaps in  $\beta$ -stable neutron star matter with electrons and muons using a relativistic Dirac-Brueckner-Hartree-Fock approach, starting with modern meson-exchange models for the nucleon-nucleon interaction. Results are given for superconducting  $^1S_0$  protons and  $^3P_2$  and  $^1D_2$  neutron superfluids. A comparison is made with recent non-relativistic calculations and the implications for neutron star cooling are discussed.

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Superfluidity and superconductivity of matter in neutron stars is expected to have a number of consequences directly related to observation, see Refs. [1–5]. Among processes that will be affected are the emission of neutrinos. Neutrino emission from e.g. various URCA processes are expected to be the dominant cooling mechanism in neutron stars less than  $10^5 - 10^6$  years old. Typically, proton superconductivity reduces considerably the energy losses in so-called modified URCA processes and may have important consequences for the cooling of young neutron stars. Another possible manifestation of superfluid phenomena in neutron stars is glitches in rotational rates observed in a number of pulsars. Moreover, the estimation of superfluid gaps and studies of pairing are not only important issues in neutron star matter, but also in the rapidly developing field of neutron-rich systems such as heavy nuclei close to the neutron drip line [6] or the study of light halo nuclei [7]. Therefore, theoretical studies of pairing in neutron-rich assemblies form currently a central issue in nuclear physics and nuclear astrophysics.

The aim of this Letter is to present results from self-consistent calculations for neutron and proton pairing gaps in  $\beta$ -stable matter relevant for neutron star studies. Pairing in the partial waves  $^1S_0$ ,  $^3P_2$  and  $^1D_2$  will be studied using a relativistic approach with modern meson-exchange potential models to describe the nucleon-nucleon (NN) potential. A comparison with the corresponding non-relativistic approach is also made.

Our computational scheme is as follows:

The first ingredient in our calculation is the self-consistent evaluation of single-particle energies in  $\beta$ -stable matter starting from the meson-exchange potential models of the Bonn group [8]. These single-particle energies are obtained within the framework of the Dirac-Brueckner-Hartree-Fock (DBHF) scheme [9–11], using a medium renormalized NN potential  $G$  defined through the solution of the  $G$ -matrix equation

$$G(\omega) = V + VQ \frac{1}{\omega - QH_0Q} QG(\omega), \quad (1)$$

where  $\omega$  is the unperturbed energy of the interacting nucleons,  $V$  is the free NN potential,  $H_0$  is the unperturbed energy of the intermediate scattering states, and  $Q$  is the Pauli operator which prevents scattering into occupied states. Only ladder diagrams with two-particle states are included in Eq. (1). In this work we solve Eq. (1) using the Bonn A potential defined in Table A.2 of Ref. [8]. This potential model employs the Thompson [10,12] reduction of the Bethe-Salpeter equation, and is tailored for relativistic nuclear structure calculations. For further details, see Refs. [8,10,11].

The DBHF is a variational procedure where the single-particle energies are obtained through an iterative self-consistency scheme. To obtain the relativistic single-particle energies we solve the Dirac equation for a nucleon in the nuclear medium, with  $c = \hbar = 1$ ,

$$(\not{p} - m + \Sigma(p))\tilde{u}(p, s) = 0, \quad (2)$$

where  $m$  is the free nucleon mass and  $\tilde{u}(p, s)$  is the Dirac spinor for positive energy solutions, with  $p = (p^0, \mathbf{p})$  being a four momentum and  $s$  the spin projection. The self-energy  $\Sigma(p)$  for nucleons can be written as

$$\Sigma(p) = \Sigma_S(p) - \gamma_0 \Sigma^0(p) + \boldsymbol{\gamma} \mathbf{p} \Sigma^V(p). \quad (3)$$

Since  $\Sigma^V \ll 1$  [10,13], we approximate the self-energy by

$$\Sigma \approx \Sigma_S - \gamma_0 \Sigma^0 = U_S + U_V, \quad (4)$$

where  $U_S$  is an attractive scalar field and  $U_V$  is the time-like component of a repulsive vector field. The Dirac spinor reads then

$$\tilde{u}(p, s) = \sqrt{\frac{\tilde{E}_p + \tilde{m}}{2\tilde{m}}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{\tilde{E}_p + \tilde{m}} \chi_s \end{pmatrix}, \quad (5)$$

where  $\chi_s$  is the Pauli spinor and terms with tilde like  $\tilde{E}_p = \sqrt{\mathbf{p}^2 + \tilde{m}^2}$  represent medium modified quantities. Here we have defined [10,13]  $\tilde{m} = m + U_S$ . In all equations below, a momentum  $p$  refers to the three-momentum  $\mathbf{p}$ . The single-particle energies  $\tilde{\epsilon}_p$  can then be written as

$$\tilde{\epsilon}_p = \tilde{E}_p + U_V. \quad (6)$$

The single-particle potential  $u_p$  is given by  $u_p = U_S \tilde{m} / \tilde{E}_p + U_V$  and can in turn be defined in terms of the  $G$ -matrix

$$u_p = \sum_{h \leq k_F} \frac{\tilde{m}^2}{\tilde{E}_h \tilde{E}_p} \langle ph | G(\omega = \tilde{\epsilon}_p + \tilde{\epsilon}_h) | ph \rangle, \quad (7)$$

where  $ph$  represent quantum numbers like momentum, spin, isospin projection etc of the different single-particle states and  $k_F$  is the Fermi momentum. Eqs. (6)–(7) are solved self-consistently starting with adequate values for the scalar and vector components  $U_S$  and  $U_V$ . The proton fraction in  $\beta$ -equilibrium is determined by imposing the relevant equilibrium conditions on the processes  $e^- + p \rightarrow n + \nu_e$  and  $e^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_e$ . The conditions for  $\beta$ -equilibrium require that  $\mu_n = \mu_p + \mu_e$ , where  $\mu_i$  is the chemical potential of particle species  $i$ , and that charge is conserved  $n_p = n_e$ , where  $n_i$  is the particle number density for particle species  $i$ . We also include muons and the condition for charge conservation becomes  $n_p = n_e + n_\mu$ , and chemical equilibrium gives  $\mu_e = \mu_\mu$ . Throughout we have assumed that neutrinos escape freely from the neutron star. The proton and neutron chemical potentials are determined from the energy per baryon, calculated self-consistently in the above DBHF approach.

The next step in our calculations is to evaluate the pairing gaps for various partial waves. To evaluate the pairing gap we follow the scheme of Baldo *et al.* [14], originally proposed by Anderson and Morel [15]. These authors introduced an effective interaction  $\tilde{V}_{k,k'}$ . This effective interaction sums up all two-particle excitations above a cutoff momentum  $k_M$ ,  $k_M = 3 \text{ fm}^{-1}$  in this work. It is defined according to

$$\tilde{V}_{k,k'} = V_{k,k'} - \sum_{k'' > k_M} V_{k,k''} \frac{1}{2\mathcal{E}_{k''}} \tilde{V}_{k'',k'}, \quad (8)$$

where the energy  $\mathcal{E}_k$  is given by  $\mathcal{E}_k = \sqrt{(\tilde{\epsilon}_k - \tilde{\epsilon}_F)^2 + \Delta_k^2}$ ,  $\tilde{\epsilon}_F$  being the single-particle energy at the Fermi surface,  $V_{k,k'}$  is the free nucleon-nucleon potential in momentum space, defined by the three-momenta  $k, k'$ . The renormalized potential  $\tilde{V}_{k,k'}$  and the free NN potential  $V_{k,k'}$  carry a factor  $\tilde{m}^2 / \tilde{E}_k \tilde{E}_{k'}$ , due to the normalization chosen for the Dirac spinors in nuclear matter. These constants are also included in the evaluation of the  $G$ -matrix, as discussed in [10,11]. For the  $^1S_0$  channel, the pairing gap  $\Delta_k$  is [16]

$$\Delta_k = - \sum_{k' \leq k_M} \tilde{V}_{k,k'} \frac{\Delta_{k'}}{2\mathcal{E}_{k'}}. \quad (9)$$

For the  $^3P_2$  partial wave we employ the expressions given in Ref. [17], modified as well by the above normalization constants. For further details, see e.g. Refs. [14,17,18]. In summary, first we obtain the self-consistent DBHF single-particle spectrum  $\tilde{\epsilon}_k$  for protons and neutrons in  $\beta$ -stable matter using the method detailed in Ref. [18]. Thereafter we solve self-consistently Eqs. (8) and (9) in order to obtain the pairing gap  $\Delta$  for protons and neutrons for different partial waves.

Our results for the pairing gaps, scalar and vector potentials for neutrons and protons, proton and neutron fractions and the chemical potential for electrons (and muons for total baryonic densities greater than  $\rho = 0.15 \text{ fm}^{-3}$ ) are displayed in Tables I and II as functions of the total baryonic density. The results of these tables can in turn be used in relativistic equations for various modified URCA processes, in a similar way as done in the non-relativistic approach of Friman and Maxwell [20]. In Fig. 1 we plot as function of the total baryonic density the pairing gap for protons in the  $^1S_0$  state, together with the results from the non-relativistic approach discussed in Refs. [18,19]. The results in the latter references were also obtained with the Bonn A potential of Ref. [8]. These results are all for matter in  $\beta$ -equilibrium. In Fig. 2 we plot the corresponding relativistic results for neutron energy gaps in the  $^3P_2$  channel. For the  $^1D_2$  channel we found both the non-relativistic and the relativistic energy gaps to vanish. The non-relativistic results for the Bonn A potential are taken from Ref. [17]. We have omitted a discussion on neutron

pairing gaps in the  $^1S_0$  channel, since these appear at densities corresponding to the crust of the neutron star. The gap in the crustal material is unlikely to have any significant effect on cooling processes [2].

As can be seen from Fig. 1, there are only small differences (except for higher densities) between the non-relativistic and relativistic proton gaps in the  $^1S_0$  wave. This is expected, since the proton fractions (and their respective Fermi momenta) are rather small, see Table I. For neutrons however, see Table II, the Fermi momenta are larger, and we would expect relativistic effects to be important. At Fermi momenta which correspond to the saturation point of nuclear matter,  $k_F = 1.36 \text{ fm}^{-1}$  the lowest relativistic correction to the kinetic energy per particle is of the order of 2 MeV. At densities higher than the saturation point, relativistic effects should be even more important, as can clearly be seen in the calculations of Ref. [10]. Since we are dealing with very small proton fractions in Table II, a Fermi momentum of  $k_F = 1.36 \text{ fm}^{-1}$ , would correspond to a total baryonic density  $\sim 0.09 \text{ fm}^{-3}$ . Thus, at larger densities relativistic effects for neutrons should be important. This is also reflected in Fig. 2 for the pairing gap in the  $^3P_2$  channel. The relativistic  $^3P_2$  gap is more than half less than the corresponding non-relativistic one, and the density region is also much smaller. This is mainly due to the inclusion of relativistic single-particle energies in the energy denominator of Eq. (9) and the normalization factors for the Dirac spinors in the NN potential. Even the non-relativistic energy gaps for neutron star matter in  $\beta$ -equilibrium are small compared with the results for pure neutron matter, where the  $^3P_2$  energy gap has a maximum around  $\sim 0.12 - 0.13 \text{ MeV}$ , see Refs. [17,21,22]. The consequences for cooling rates and the interior composition of a neutron star are significant. A recent investigation of various cooling mechanisms by Schaab *et al.* [23] found that an agreement with observed surface temperatures was reached if the  $^3P_2$  energy gaps were of the order  $\sim 0.05 \text{ MeV}$ . Our non-relativistic results for  $\beta$ -stable matter are of this size, while the relativistic energy gaps result in an almost negligible suppression of e.g. various modified URCA processes in the interior of a neutron star. These results, and those of Schaab *et al.* [23] as well, are at askance with those of Page [4], where, in order to explain the observed temperature of Geminga, baryon pairing has to be present in most, if not all of the core of the star.

In summary, in this work we have calculated in a self-consistent way single-particle energies and energy gaps using a relativistic DBHF approach. To our knowledge, after the relativistic work of Kucharek and Ring [16], this is the first estimate of pairing gaps within the framework of the DBHF approach. In Ref. [16], the  $^1S_0$  gap in symmetric nuclear matter was studied within the framework of the Serot-Walecka model [13]. The only parameters which enter our approach are those which define the free NN potential [8]. Here we have focused on pairing in dense matter, though our approach allows also for a consistent treatment of other neutron star properties. The same NN force used here has also been used in Ref. [24] to calculate the equation of state and the total mass and radius for a neutron star. Combining the results from this work and those of Refs. [19,24], the following picture emerges:

Within the DBHF approach, the direct URCA processes are only allowed for densities larger than  $0.52 \text{ fm}^{-3}$ , see Ref. [19]. A neutron star with total mass  $1.6M_\odot$  would have a central density of  $\rho_c = 0.4 \text{ fm}^{-3}$  in  $\beta$ -stable matter [24]. For such a central density, various modified URCA processes are possible mechanisms for neutrino production in a neutron star. The main suppression of these processes would then come from protons in the  $^1S_0$  state. The reader should note that there are other possible cooling mechanisms than those discussed here, such as neutrino-pair bremsstrahlung [25], direct URCA with hyperons or neutrino emissions from more exotic states, such as pion and kaon condensates or quark matter, see e.g. Refs. [4,5,23] for recent reviews. However, for a star with central density  $\rho_c = 0.4 \text{ fm}^{-3}$ , many of these more exotic neutrino emissivities are less likely. Hyperons appear at densities  $\rho \sim 0.3$  or greater [26]. Similar densities are expected for kaons and quark matter [23,26]. In addition, neutrino-pair bremsstrahlung was recently found [25] to be much less important than previously estimated. Thus, for a  $1.6M_\odot$  neutron star with central density of  $0.4 \text{ fm}^{-3}$  obtained with our DBHF approach [24], the most likely cooling scenario is through modified URCA processes, and the main suppression comes from superconducting protons in the  $^1S_0$  state. Finally, it ought to be noted that we have not included effects from medium polarization effects, as discussed in Ref. [27]. These may further change the size of the energy gaps and the neutrino emissivities (the  $^1S_0$  gap should decrease while the  $^3P_2$  gap is expected to increase). Further, we have not considered the possibility of  $^3D_2$  pairing, which appear due to the increased proton fraction, as discussed by Alm *et al.* [28].

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TABLE I. Proton fractions  $\chi_p$ , scalar and vector single-particle potentials  $U_S^p$  and  $U_V^p$ , respectively, for protons, the proton pairing gap  $\Delta_p$  for protons in the  $^1S_0$  state and the electron (and muon) chemical potential  $\mu_e$  as functions of total baryonic density  $\rho$ . Densities are in units of  $\text{fm}^{-3}$ ,  $U_S^p$ ,  $U_V^p$ ,  $\Delta_p$  and  $\mu_e$  in units of MeV.

$\rho$	$\chi_p$	$U_S^p$	$U_V^p$	$\Delta_p$	$\mu_e$
.0013	.0032	-7.8479	3.2471	.0121	11.7231
.0068	.0050	-77.7002	61.7252	.0483	20.3904
.0281	.0096	-172.0541	135.3744	.2024	38.9884
.0583	.0156	-236.5725	181.5207	.4386	58.5459
.0944	.0229	-285.0128	213.1141	.7036	78.1881
.1377	.0307	-329.1642	242.7944	.9107	98.3550
.1811	.0403	-365.8355	270.4411	1.0160	115.8907
.2007	.0462	-381.3338	283.5829	1.0173	123.0215
.2212	.0524	-396.7707	297.3635	.9742	130.1985
.2627	.0658	-424.5634	325.2710	.7712	143.9456
.3072	.0801	-451.9637	357.1098	.4490	158.2441
.3304	.0877	-464.7640	373.9551	.2638	165.5386
.3544	.0953	-476.8407	391.2967	.1826	172.9228
.3594	.0968	-479.2122	394.8924	.0856	174.4599

TABLE II. Proton fraction  $\chi_p$ , neutron scalar and vector single-particle potentials  $U_S^n$  and  $U_V^n$ , respectively and the neutron pairing gap  $\Delta(^3P_2)$  as functions of total baryonic density  $\rho$ . Densities are in units of  $\text{fm}^{-3}$ ,  $U_S^n$ ,  $U_V^n$  and  $\Delta$  in units of MeV.

$\rho$	$\chi_p$	$U_S^n$	$U_V^n$	$\Delta(^3P_2)$
.0756	.0191	-118.4076	90.1259	0.009
.0811	.0202	-127.8562	97.9057	0.013
.0849	.0210	-134.2159	103.1913	0.014
.0949	.0230	-150.7538	116.9925	0.017
.1012	.0243	-161.1272	125.6867	0.017
.1056	.0252	-167.9521	131.2468	0.017
.1125	.0266	-179.0345	140.6626	0.015
.1172	.0275	-186.6448	147.1867	0.013
.1196	.0279	-190.5106	150.5173	0.011

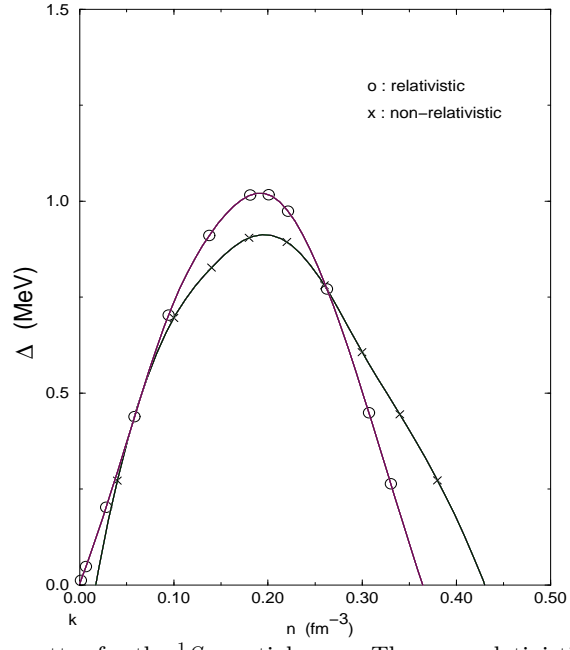


FIG. 1. Proton pairing in  $\beta$ -stable matter for the  $^1S_0$  partial wave. The non-relativistic results are taken from Ref. [18].

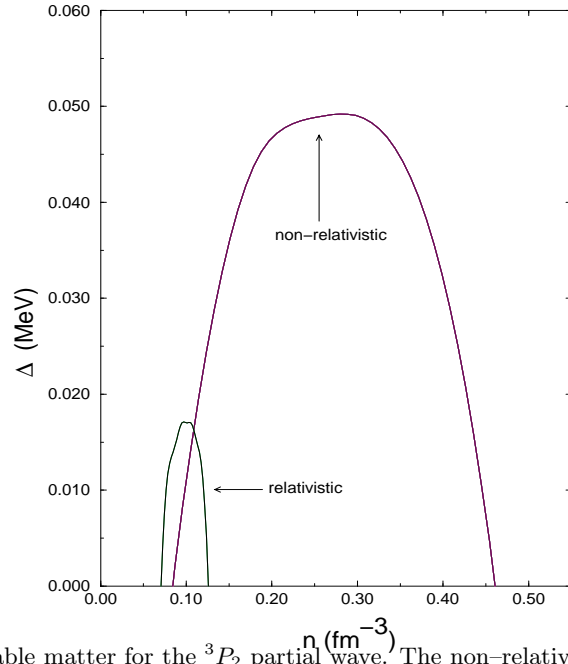


FIG. 2. Neutron pairing in  $\beta$ -stable matter for the  $^3P_2$  partial wave. The non-relativistic results are taken from Ref. [17].